BC Calc - Derivative Practice: All Rules

① $f(t) = \dfrac{t^2}{\sqrt{t+1}} = t^2 \cdot (t+1)^{-1/2}$

$f'(t) = -\frac{1}{2}t^2(t+1)^{-3/2} + 2t(t+1)^{-1/2}$

$= (t+1)^{3/2}\left(-\frac{1}{2}t^2 + 2t(t+1)\right)$

$= \dfrac{\frac{3}{2}t^2 + 2t}{(t+1)^{3/2}}$

② $f(x) = \dfrac{x^2+1}{x^3} = x^{-1} + x^{-3}$

$f'(x) = -x^{-2} - 3x^{-4}$

$= \dfrac{-1}{x^2} - \dfrac{3}{x^4}$

$= \dfrac{-(x^2+3)}{x^4}$

③ $z = (x+1)^3(5-x)^4$

$\dfrac{dz}{dx} = 3(x+1)^2(5-x)^4 - 4(x+1)^3(5-x)^3$

$= (x+1)^2(5-x)^3\left[3(5-x) - 4(x+1)\right]$

$= (x+1)^2(5-x)^3(11-7x)$

④ $f(\theta) = \dfrac{1}{\tan(2\theta)} = \cot(2\theta)$

$f'(\theta) = -2\csc^2(2\theta)$

⑤ $f(x) = 3x \cdot 2^{5x}$

$f'(x) = 3 \cdot 2^{5x} + 3x \cdot 5 \cdot 2^{5x} \cdot \ln 2$

$= 3 \cdot 2^{5x}(1 + 5x \cdot \ln 2)$

$f''(x) = 15 \ln 2 \cdot 2^{5x}(2 + 5x\ln 2)$

⑥ $f(\beta) = \dfrac{\beta y + y^6}{1-\beta}$

y is a constant

$f'(\beta) = \dfrac{y(1-\beta) + \beta y + y^6}{(1-\beta)^2}$

$= \dfrac{y + y^6}{(1-\beta)^2}$

⑦ $y = \ln(\ln(2t^3))$

$\dfrac{dy}{dx} = \dfrac{1}{\ln(2t^3)} \cdot \dfrac{1}{2t^3} \cdot 6t^2$

$= \dfrac{3}{t \cdot \ln(2t^3)}$

⑧ $g(x) = x \cdot e^{x^2}$

$g'(x) = e^{x^2} + x \cdot e^{x^2} \cdot 2x$

$= e^{x}(1 + 2x^2)$

(9) $x(r) = 3\sqrt[3]{r} - \sqrt{\frac{3}{r}} + \frac{1}{3r}$

$= 3(r)^{1/3} - \sqrt{3}\cdot r^{-1/2} + \frac{1}{3}r^{-1}$

$x'(r) = r^{-2/3} + \frac{1}{2}\sqrt{3}\cdot r^{-3/2} - \frac{1}{3}r^{-2}$

$= \dfrac{1}{\sqrt[3]{r^2}} + \dfrac{\sqrt{3}}{2\sqrt{r^3}} - \dfrac{1}{3r^2}$

(10) $h(x) = \dfrac{\cos x}{1 - \sin x}$

$h'(x) = \dfrac{-\sin x(1-\sin x) + \cos x \cdot \cos x}{(1-\sin x)^2}$

$= \dfrac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$

$= \dfrac{-\sin x + 1}{(1-\sin x)^2} = \dfrac{1}{1-\sin x}$

(11) $z = 10^{2\log_{10} x}$

$= 10^{\log_{10} x^2} = x^2$

$\dfrac{dz}{dx} = 2x$

(12) $f(x) = \sec^{-1}(4x^2+1)$

$f'(x) = \dfrac{8x}{|4x^2+1|\sqrt{(4x^2+1)^2 - 1}}$

$= \dfrac{8x}{(4x^2+1)\sqrt{16x^4 + 8x^2}}$

$= \dfrac{8x}{(4x^2+1)(2x)\sqrt{4x^2+2}} = \dfrac{4}{(4x^2+1)\sqrt{4x^2+2}}$

(13) $f(t) = \tan^{-1}\left(\frac{2}{t}\right)$

$f'(t) = \dfrac{1}{1 + \left(\frac{2}{t}\right)^2} \cdot -2t^{-2}$

$= \dfrac{-2}{t^2\left(1 + \frac{4}{t^2}\right)}$

$= \dfrac{-2}{t^2+4}$

(14) $g(\theta) = (3\theta + \tan^2(4\theta))^{1/2}$

$g'(\theta) = \frac{1}{2}(3\theta + \tan^2(4\theta))^{-1/2}(3 + 2\tan(4\theta)\cdot 4\cdot \sec^2 4\theta)$

$= \dfrac{3 + 8\tan(4\theta)\sec^2(4\theta)}{2\sqrt{3\theta + \tan^2(4\theta)}}$

(15) $f(x) = x \cdot \cos(e^x)$

$f'(x) = \cos(e^x) - x \cdot \sin(e^x) \cdot e^x$

(16) $x^2 - y^2 = 25$

$2x - 2y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

$y'' = \frac{y - xy'}{y^2} = \frac{y - x\left(\frac{x}{y}\right)}{y^2}\left(\frac{y}{y}\right)$

$y'' = \frac{y^2 - x^2}{y^3}$

(17) $g(z) = (\sin z)^{2z+1}$

$\ln(g(z)) = (2z+1) \cdot \ln(\sin z)$

$\frac{1}{g(z)} \cdot g'(z) = 2 \cdot \ln(\sin z) + \frac{(2z+1)\cos z}{\sin z}$

$g'(z) = (2 \cdot \ln(\sin z) + (2z+1) \cdot \cot z) \cdot (\sin z)^{2z+1}$

(19) $a(t) = \ln\left(\frac{1 - \cos t}{1 + \cos t}\right)^4$

$= 4 \cdot \ln(1 - \cos t) - 4 \cdot \ln(1 + \cos t)$

$a'(t) = 4 \cdot \frac{1}{1 - \cos t} \cdot \sin t + 4 \cdot \frac{1}{1 + \cos t} \cdot \sin t$

$\left(\frac{1+\cos t}{1+\cos t}\right)\frac{4 \cdot \sin t}{1 - \cos t} + \frac{4 \cdot \sin t}{1 + \cos t}\left(\frac{1 - \cos t}{1 - \cos t}\right)$

$\frac{4\sin t + 4\sin t \cdot \cos t + 4\sin t - 4\sin t \cdot \cos t}{1 - \cos^2 x}$

$= \frac{8 \sin t}{\sin^2 t}$

$= \frac{8}{\sin t} = 8 \csc t$

(18) $f(x) = \frac{x^2}{(2+x)^3}$

$f'(x) = \frac{2x(2+x)^3 - 3 \cdot x^2 (2+x)^2}{(2+x)^6}$

$= \frac{x(2+x)^2(4 + 2x - 3x)}{(2+x)^6}$

$= \frac{4x - x^2}{(2+x)^4}$

(20) $g(1) = 2(1)^3 - (1)^2 + 1 = 2$

$(g^{-1})'(2) = \frac{1}{g'(1)}$

$= \frac{1}{6(1)^2 - 2(1)}$

$= \frac{1}{4}$